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SOLUTIONS OF PROBLEMS.

2669. Proposed by S. A. COREY, Albia, Iowa.

Let A_1, A_2, $\cdots$, A_8, and $-(A_1 + A_2 + \cdots + A_8)$ be the vector sides of an enneagon, plane or gauche. Also let B_1, B_2, $\cdots$, B_8, and $-(B_1 + B_2 + \cdots + B_8)$ be the vector sides of a second enneagon, where $B_1 = C_1A_1 - C_2C_5A_3 - C_3C_6A_5 + C_4C_5C_6A_7$, $B_2 = C_1A_2 - C_2C_5A_4 - C_3C_6A_6 + C_4C_5C_6A_8$, $B_3 = C_2A_1 + C_1A_3 - C_4C_6A_5 - C_3C_6A_7$, $B_4 = C_2A_2 + C_1A_4 - C_4C_6A_6 - C_3C_6A_8$, $B_5 = C_3A_1 + C_4C_5A_3 + C_1A_5 + C_2C_5A_7$, $B_6 = C_3A_2 + C_4C_5A_4 + C_1A_6 + C_2C_5A_8$, $B_7 = C_4A_1 - C_3A_3 + C_2A_5 - C_1A_7$, and $B_8 = C_4A_2 - C_3A_4 + C_2A_6 - C_1A_8$, C_1, C_2, C_3, C_4, C_5, and C_6 being scalars.

Then, if a_s = tensor A_s, b_s = tensor B_s, and $\cos(A_rA_s)$ = cosine of the angle included between A_r and A_s and $\cos(B_rB_s)$ = cosine of the angle included between B_r and B_s, establish the following relation between the sides and angles of the two enneagons:

$$[C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2][a_1a_2\cos(A_1A_2) + C_5a_3a_4\cos(A_3A_4) + C_6a_5a_6\cos(A_5A_6) + C_5C_6a_7a_8\cos(A_7A_8)]$$
$$= b_1b_2\cos(B_1B_2) + C_5b_3b_4\cos(B_3B_4) + C_6b_5b_6\cos(B_5B_6) + C_5C_6b_7b_8\cos(B_7B_8).$$

Show that Geometry problem 506 is a special case of the foregoing. Give illustrative example, using triangle or other simple geometric figure, by assuming that some of the sides of the first enneagon are zero.

Solution by Proposer.

Whenever A_1, A_2, $\cdots$ and A_8 are scalar (or algebraic) quantities, and B_1, B_2, $\cdots$ and B_8 have scalar values corresponding in form to those given in the problem, we have the algebraic identity,

$$(C_1^2 + C_5C_2^2 + C_6C_3^2 + C_5C_6C_4^2)(A_1A_2 + C_5A_3A_4 + C_6A_5A_6 + C_5C_6A_7A_8)$$
$$= B_1B_2 + C_5B_3B_4 + C_6B_5B_6 + C_5C_6B_7B_8.$$

Inasmuch as all the terms in A_1, A_2, $\cdots$ A_8, and B_1, B_2, $\cdots$ B_8 in this algebraic identity are of the second degree, a geometric interpretation may be obtained by assuming that A_1, A_2, $\cdots$ A_8 and B_1, B_2, $\cdots$ B_8, are vectors. This follows immediately from the fact that vector multiplication is commutative in so far as the scalar part of the product is concerned whenever all the vector terms employed are of the second degree. But the scalar part of the vector product B_rB_s is $-b_rb_s\cos(B_rB_s)$. Substituting this scalar part of the vector product in both members of the above algebraic equation and changing signs we obtain at once the equation contained in the problem. If $A_1 = A_2$, $A_3 = A_4$, $A_5 = A_6$, and $A_7 = A_8$, the problem becomes identical with Geometry problem 506.

Example. As long as a vector maintains a constant length and direction in space, its origin in space may be altered at will. Hence we need not confine our attention to closed geometric figures in interpreting the given identity. Let DEF be a given triangle. Bisect DE in G. Draw GF and extend EF to F'. Draw GH intersecting DF in I and EF' in J. Let $A_1 = GF$, $A_2 = GI$, and $A_3 = GD$. Also let $A_4 = A_5 = A_6 = A_7 = A_8 = 0$, $C_1 = C_2 = C_3 = C_4 = C_5 = 1$, and $C_6 = 0$. Substituting in the given identity, paying strict attention to the direction of the vectors employed, and dividing by the constant factor, we readily get $2GF\cos FGI = DF\cos GID + EF\cos EJG$, a known result.

2670. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A telegraph wire, weighing one tenth pound per yard, is stretched between poles on level ground, so that the greatest dip of the wire is three feet. Find approximately the distance between the poles when the tension at the lowest point of the wire is 140 pounds.

Solution by Elbert H. Clarke, Hiram College, Ohio.

It is a well-known property of wires hanging freely from two supports that the tension at any point is equal to that which would be produced by a wire of the same weight per unit length, hanging vertically and being of the same length as the ordinate of the point where the tension is measured, the equation of the curve being of the form,

$$y = \frac{a}{2}(e^{x/a} + e^{-x/a}).$$

Hence, the equation of the catenary curve assumed by the wire in the above problem will be

$$y = 700(e^{x/1400} + e^{-x/1400}),$$

the unit of distance being one yard.

The amount of dip or sag in a symmetrical segment of horizontal projection, $2h$, will be

$$d = \frac{a}{2}(e^{h/a} + e^{-h/a}) - a$$

$$= \frac{h^2}{2a} + \frac{h^4}{24a^3} + \frac{h^6}{720a^5} + \cdots.$$

In our problem $d = 1$, $a = 1400$, and a very close first approximation is given by neglecting all terms in the series after the first. Hence, $h = 52.915$, approximately. Using more terms the result correct to three places is $h = 52.912$. The distance between the poles, correct to the nearest tenth of a foot is 317.5 feet.

Also solved by ROGER A. JOHNSON.

2671. Proposed by ARTEMAS MARTIN, Washington, D. C.

Find two rectangular parallelopipedons whose edges are rational whole numbers and whose solid diagonals are also rational whole numbers and equal.

SOLUTION BY S. A. COREY, Albia, Iowa.

We have the identities

$$\begin{aligned}(x^2 + y^2 + u^2 + v^2)^2 &= (x^2 - y^2 - u^2 + v^2)^2 + (2xy - 2uv)^2 + (2ux + 2vy)^2\\ &= (x^2 - y^2 + u^2 - v^2)^2 + (2xy + 2uv)^2 + (2vx - 2uy)^2\\ &= (x^2 + y^2 - u^2 - v^2)^2 + (2ux - 2vy)^2 + (2vx + 2uy)^2.\end{aligned}$$

By letting x, y, u, and v represent rational whole numbers any number of solutions of the problem may be obtained. One such solution is obtained by letting $x = 1$, $y = 2$, $u = 5$, and $v = 7$, and we find *three* parallelopipedons fulfilling the requirements of the problem, with edges 21, 66, 38; 27, 74, 6; and 69, 18, 34, respectively, the solid diagonal of each being 79. The three smallest have edges, 1, 2, 2; 2, 4, 4; and 2, 3, 6, respectively.

Also solved by L. E. LUNN, W. P. DURFEE, and H. L. OLSON.

2673. Proposed by WILLIAM O. BEAL, University of Minnesota.

A plane through the center of an oblate spheroid makes an angle, i, with the plane of its equator. Express the eccentricity, e', of this section in terms of the eccentricity, e, of a meridian section and the angle, i.

SOLUTION BY C. A. BARNHART, Colorado College.

Let the equation of the oblate spheroid be $x^2/b^2 + y^2/b^2 + z^2/a^2 = 1$, $(b > a)$, of which the z-axis is the axis of revolution. Let $OP = b'$ be the semi-minor axis of the given plane section of which b is the semi-major axis. Then P will be a point on the meridian section, which may be assumed to lie in the yz-plane and to have the parametric equations:

$$z = a \sin \theta, \qquad y = b \cos \theta;$$

and the eccentricity,

$$e = \frac{\sqrt{b^2 - a^2}}{b}.$$